$\square$ 10MAT31

Third Semester B.E. Degree Examination, June/July 2018
Engineering Mathernatics - III
Time: 3 hrs.

## PART - A

1 a. Obtain the Fourier Series for the function,
$f(x)=\left\{\begin{array}{cc}\pi x & \text { in } 0 \leq x \leq 1 \\ \pi(2-x) & \text { in } 1 \leq x \leq 2\end{array}\right.$.
(07 Marks)
b. Find the cosine half range series for $\mathrm{f}(\mathrm{x})=\mathrm{x}(l-\mathrm{x}) ; 0 \leq \mathrm{x} \leq l$.
(06 Marks)
c. Obtain the Fourier series of $y$ upto the second harmonics for the following values:
(07 Marks)

| $\mathrm{x}^{0}$ | 45 | 90 | 135 | 180 | 225 | 270 | 315 | 360 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y | 4.0 | 3.8 | 2.4 | 2.0 | -1.5 | 0 | 2.8 | 3.4 |

## > Note: Answer FIVE fuill questions, selecting at least TWO questions from each part. <br> <br> Note: Answer FIVE fiili questions, selecting <br> <br> Note: Answer FIVE fiili questions, selecting at least TWO questions from each part.

 at least TWO questions from each part.}2 a. Find the Fourier transform of $f(x)=e^{-|x|}$.
(07 Marks)
b. Find the Fourier sine transform of $f(x)=\frac{1}{x\left(1+x^{2}\right)}$.
(06 Marks)
c. Find the Fourier cosine transform of $\mathrm{e}^{-\mathrm{ax}}$ and deduce that $\int_{0}^{\infty} \frac{\cos m x}{a^{2}+x^{2}} d x=\frac{\pi}{2 a} e^{-a m}$.
(07 Marks)

3 a. Obtain the various possible solution of one-dimensional wave equation $u_{t t}=C^{2} u_{x x}$ by the method of separation of variables.
(07 Marks)
b. A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If each of its points is given a velocity $\lambda x(l-x)$. Find the displacement of the string at any distance $x$ from one end at any time $t$.
(06 Marks)
c. Solve the Laplace equation, $\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}=0$
subject to the conditions $u(0, y)=u(l, y)=u(x, 0)=0$ and $u(x, a)=\sin \frac{n \pi x}{l}$.
(07 Marks)
4 a. Predict the mean radiation dose at an altitude of 3000 feet by fitting an exponential curve to the given data using $y=a b^{x}$
(07 Marks)

| Altitude (x): | 50 | 450 | 780 | 1200 | 4400 | 4800 | 5300 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Dose of radiation $(\mathrm{y}):$ | 28 | 30 | 32 | 36 | 51 | 58 | 69 |

b. Using graphical method solve the LPP,

Maximize $\mathrm{z}=50 \mathrm{x}_{\mathrm{y}}+60 \mathrm{x}_{2}$,
Subject to the constraints: $2 \mathrm{x}_{1}+3 \mathrm{x}_{2} \leq 1500$,

$$
\begin{aligned}
& 3 x_{1}+2 x_{2} \leq 1500 \\
& 0 \leq x_{1} \leq 400 \\
& 0 \leq x_{2} \leq 400 \\
& x_{1} \geq 0, x_{2} \geq 0
\end{aligned}
$$

(06 Marks)
c. Solve the following minimization problem by simplex method:

Objective function: $P=-3 x+8 y-5 z$
Constraints : $-x-2 z \leq 5$,

$$
\begin{gathered}
2 x-3 y+z \leq 3 \\
2 x-5 y+6 z \leq 5 \\
x_{1}, x_{2}, x_{3} \geq 0
\end{gathered}
$$

(07 Marks)

## PART - B

5 a. Using Newton-Raphson iterative formula find the real root of the equation $\mathrm{x} \log _{10} \mathrm{x}=1.2$. Correct to five decimal places.
(07 Marks)
b. Solve, by the relaxation method, the following system of equations:
$9 x-2 y+z=50$
$x+5 y-3 z=18$
$-2 \mathrm{x}+2 \mathrm{y}+7 \mathrm{z}=19$.
(06 Marks)
c. Using the Rayleigh's power method find the dominant eigen value and the corresponding eigen vector of the matrix, $A=\left[\begin{array}{ccc}2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2\end{array}\right]$ taking $[1,1,1]^{\mathrm{T}}$ as the initial eigen vector.
Peform five iterations.
(07 Marks)

6 a. The population of a town is given by the table. Using Newton's forward and backward interpolation formulae, calculate the increase in the population from the year 1955 to 1985.
(07 Marks)

| Year | 1951 | 1961 | 1971 | 1981 | 1991 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Population in thousands | 19.96 | 39.65 | 58.81 | 77.21 | 94.61 |

b. The observed values of a function are respectively 168, 120, 72 and 63 at the four positions $3,7,9,10$ of the independent variable. What is the best estimate you can give for the value of the function at the position 6 of the independent variable? Use Lagrange's method.
(06 Marks)
c. Use Simpson's $\left(\frac{3}{8}\right)^{\text {th }}$ Rule to obtain the approximate value of $\int_{0}^{0.3}\left(1-8 x^{3}\right)^{\frac{1}{2}} \mathrm{dx}$ by considering 3 equal intervals.
(07 Marks)

7 a. Solve numerically the wave equation $\mathrm{u}_{\mathrm{xx}}=0.0625 \mathrm{u}_{\mathrm{tt}}$ subject to the conditions, $u(0, t)=0=u(5, t), u(x, 0)=x^{2}(x-5)$ and $u_{t}(x, 0)=0$ by taking $h=1$ for $0 \leq t \leq 1$.
(07 Marks)
b. Solve : $\mathrm{u}_{\mathrm{xx}}=32 \mathrm{u}_{\mathrm{t}}$ subject to the conditions, $\mathrm{u}(0, \mathrm{t})=0, \mathrm{u}(1, \mathrm{t})=\mathrm{t}$ and $\mathrm{u}(\mathrm{x}, 0)=0$. Find the values of tupto $\mathrm{t}=5$ by Schmidt's process taking $\mathrm{h}=\frac{1}{4}$. Also extract the following values:
(i) $\mathrm{u}(0.75,4)$
(ii) $\mathrm{u}(0.5,5)$
(iii) $\mathrm{u}(0.25,4)$
(06 Marks)
c. Solve the Laplace equation $\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}=0$ in the square region shown in the following Fig. Q7 (c), with the boundary values as indicated in the figure. Carry out two iterations.
(07 Marks)


Fig. Q7 (c)
8 a. State initial value property and final value property. If $\bar{u}(z)=\frac{2 z^{2}+3 z+4}{(z-3)^{3}},|z|>3$. Find the values of $\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}$.
b. Obtain the inverse $z$-transform of the function, $\frac{4 z^{2}-2 z}{z^{3}-5 z^{2}+8 z-4}$.
c. Solve the difference equation, $\mathrm{y}_{\mathrm{n}+1}+\frac{1}{4} \mathrm{y}_{\mathrm{n}}=\left(\frac{1}{4}\right)^{\mathrm{n}},(\mathrm{n} \geq 0), \mathrm{y}_{0}=0$ by using z -transform method.


# Third Semester B.E. Degree Examination, June/July 2018 Strength of Materials 

Time: 3 hrs.
Max. Marks:100

## Note: Answer FIVE fuil questions, selecting at least TWO fuli questions from each part.

## PART - A

1 a. Define: i) Hooke's law ani ii) Modulus of rigidity.
(04 Marks)
b. Derive an relation between modulus of rigidity, modulus of elasticity and Poisson's ratio.
(06 Marks)
c. A stepped bar is subjected to an external loading as shown in Fig.Q.1(c). Calculate the change in length of the bar. Take E for steel $=200 \mathrm{GPa}, \mathrm{E}$ for aluminium $=70 \mathrm{GPa}$ and E for copper $=100 \mathrm{GPa}$.
(05 Marks)

d. A solid alloy bar of 40 mm diameter is used as tie. If the permissible tensile stress in material is $32 \mathrm{MN} / \mathrm{m}^{2}$, determine the capacity of the bar. If a hollow steel bar with internal diameter 20 mm is used instead of solid bar, determine its external diameter if the permissible stress is $150 \mathrm{MN} / \mathrm{m}^{2}$.
(05 Marks)
2 a. Define composite section.
(02 Marks)
b. A reinforced concrete column of size $0.3 \mathrm{~m} \times 0.3 \mathrm{~m}$ contains 4 no .40 mm diameter rods and subjected to a load of 500 kN . Determine the stresses in concrete and steel if the modular ratio of steel to concrete is 15 .
(08 Marks)
c. A brass bar of 25 mm diameter is enclosed within a steel tube of internal diameter 25 mm and external diameter 50 mm , the length of the composite bar is 1 m and further the ends are rigidity held by means of rigid collars. Find the stresses induced in the materials when the temperature rises by $100^{\circ} \mathrm{C}$. Find the final stresses if the composite bar is subjected to a tensile load of 600 kN . E for steel $=200 \mathrm{GPa} ; \alpha$ for steel $=11.6 \times 10^{-6} /{ }^{\circ} \mathrm{C}, \mathrm{E}$ for brass $=100 \mathrm{GPa} ; \alpha$ for brass $=18.7 \times 10^{-6} / \mathrm{C} \mathrm{C}$.
(10 Marks)
3 a. Define principal stress and principal pianes.
(03 Marks)
b. Derive the expressions for normal and tangential stress components on any arbitrary plane which is inclined at an ' $\theta$ ' with horizontal in a two dimensional stress system. (07 Marks)
c. At a point in an elastic material the stresses on two perpendicular directions are $80 \mathrm{~N} / \mathrm{mm}^{2}$ compressive along X-direction, $60 \mathrm{~N} / \mathrm{mm}^{2}$ tensile along Y-direction with a shear stress of $40 \mathrm{~N} / \mathrm{mm}^{2}$. Find the normal and tangential stresses on a plane which is making an angle of $40^{\circ}$ with the plane on which the tensile stress acts. Also find the magnitude and direction of principal stress.
(10 Marks)

4 a. Define: i) Bending moment ii) Shear force.
(02 Marks)
b. Derive the relationship between bending moment, shear force and loading.
(04 Marks)
c. Draw the shear force and bending moment diagram with salient values for the overhanging beam loaded as shown in Fig.Q.4(c). Also locate the point of contra flexures, of any
(14 Marks)


Fig.Q.4(c)

## PART - B

5 a. Show that for a rectangular cross section shear stress distribution varies parabolically across the depth. Further show that maximum shear stress is 1.5 times average shear stress.
(06 Marks)
b. A cantilever beam 3 m long is subjected to a udl of $30 \mathrm{kN} / \mathrm{m}$ over the entire span. The allowable working stress in compression and tension is 150 MPa . If the cross section is to be of rectangular, determine the dimensions. Take the depth of the $\mathrm{c} / \mathrm{s}$ as twice the width.
(14 Marks)
6 a. Derive $\mathrm{EI} \frac{\mathrm{d}^{2} \mathrm{y}}{\mathrm{dx}^{2}}=+\mathrm{M}$ with usual notations.
(08 Marks)
b. A simply supported beam ' $A B$ ' of span $\frac{2 L}{3}$ 'has an overhang $B C$ of length $\frac{\text { ' } L \text { ' }}{3}$. The beam supports a uniform load of intensity ' $q$ ' per meter run over (Refer Fig.Q.6(b)),


Fig.Q.6(b)
Its entire length. Determine deflection and slope at free and ' C '.
(12 Marks)
7 a. State the assumptions made in theory of pure torsion.
(03 Marks)
b. Prove that a hollow shaft is stronger and stiffer than the solid shaft of the same material, length and weight.
(07 Marks)
c. A hollow steel shaft transmits 200 kW of power at 150 rpm . The total angle of twist in a length of 5 m of the shaft is $3^{\circ}$. Find the inner and outer diameters of the shaft if the permissible shear stress is 60 MPa . Take $\mathrm{G}=80 \mathrm{GPa}$.
(10 Marks)
8 a. Derive the Euler's expression for crippling load for column with one end fixed and other end hinged.
(08 Marks)
b. Determine the Euler's crippting load for the column of steel of diameter 50 mm and length 4 m with both ends hinged. Further compare the same with Rankine's formula. Take $\mathrm{E}=200 \mathrm{GPa}$, factor of safety $=3 ;$ Rankine's constants $\sigma_{\mathrm{c}}=320 \mathrm{MPa} ; \mathrm{a}=1 / 7500$. (12 Marks)


Third Semester B.E. Degree Examination, Dec.2017/Jan 2018 Surveying - I

Time: 3 hrs.
Max. Marks: 100

## Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART-A
1 a. What is surveying? Explain the basic principles of surveying.
(06 Marks)
b. Explain the broad classification of survey.
(08 Marks)
c. Discuss briefly the topographic maps, their numbering
(06 Marks)
2 a. Explain the method of direct ranging by the use of line ranger with a neat sketch. ( 06 Marks)
b. Explain the indirect methods of chaining on a sloping ground.
(06 Marks)
c. A steel tape 30 m long between end graduations at a temperature of $27^{\circ} \mathrm{C}$ under a pull of 45 N when lying on the flat. The tape is stretched over two supports between which it records 30000 m and is supported at two intermediate supports equally spaced. All the supports are at same level and the tape is allowed to sag freely between the supports. If the temperature in the field is $32^{\circ} \mathrm{C}$ and the pull on the tape is 75 N . Calculate the actual length between end graduations and equivalent length at MSL if measurements were made at an elevation of 1000 m . Area of cross section of tape $=7.0 \mathrm{~mm}^{2}$, Mass of tape $=1.60 \mathrm{~kg}$,

$$
\alpha=1.1 \times 10^{-5} \text { per }^{\circ} \mathrm{C}, \mathrm{E}=2 \times 10^{5} \mathrm{~N} / \mathrm{mm}^{2}, \text { Radius of earth }=6370 \mathrm{~km} . \quad(08 \text { Marks })
$$

3 a. With neat sketches, explain obstacles in chaining.
(08 Marks)
b. With a neat diagram, explain the working of an optical square.
(06 Marks)
c. In passing an obstacle in the form of a pond, stations $A$ and $D$ on the main line, were taken on the opposite sides of a pond. On the left of $A D$, a line $A B=200 \mathrm{~m}$ long was laid down and a second line $A C=250 \mathrm{~m}$ long was ranged on the right of AD , the points $\mathrm{B}, \mathrm{D}$ and C being in the same straight line. BD and DC were then chained and found to be 125 m and 150 m respectively. Find the length of $A D$.
(06 Marks)
4 a. Differentiate between:
(i) True meridian and magnetic meridian.
(ii) Dip and declination.
(iii) Agonic and Isogonic lines.
(05 Marks)
b. On an old map, a line was drawn to a magnetic bearing of $320^{\circ} 30^{\prime}$ when the declination was $3^{\prime} 30^{\prime} \mathrm{W}$. Find the present bearing of the line if the declination is $4^{\circ} 15^{\prime} \mathrm{E}$.
(04 Marks)
c. The following bearings were observed for a closed traverse ABCDEA. Calculate the included angles.
(10 Marks)

| LINE | AB | BC | CD | DE | EA |
| :---: | :--- | :--- | :--- | :--- | :--- |
| Bearing | $140^{\circ} 30^{\prime}$ | $80^{\circ} 30^{\prime}$ | $340^{\circ} 0^{\prime}$ | $290^{\circ} 30^{\prime}$ | $230^{\circ} 30^{\prime}$ |

## PART - B

5
a. Explain the following:
(i) Dependent and independent co-ordinates.
(ii) Bowditch rule and Transit rule.
(iii) Latitude and Departure.
(06 Marks)
b. A closed traverse PQRSP has the following bearings. At what station local attraction was suspected Determine the correct bearings.
(06 Marks)

| Line | PQ | QR | RS | SP |
| :---: | :---: | :---: | :---: | :---: |
| FB | $124^{\circ} 30^{\prime}$ | $68^{\circ} 15^{\prime}$ | $310^{\circ} 30^{\prime}$ | $200^{\circ} 15^{\prime}$ |
| BB | $304^{\circ} 30^{\prime}$ | $246^{\circ} 0^{\prime}$ | $135^{\circ} 15^{\prime}$ | $17^{\circ} 45^{\prime}$ |

c. In the following traverse ABCDE , the length and bearing of $\mathrm{E} A$ is omitted. Calculate the length and bearing of EA
(08 Marks)

| LINE | Length (m) | Bearing |
| :---: | :---: | :---: |
| AB | 204 | $87^{\circ} 30^{\circ}$ |
| BC | 226 | $20^{\circ} 20^{\circ}$ |
| CD | 187 | $280^{\circ} 0^{\circ}$ |
| DE | 192 | $210^{\circ} 3^{\prime}$ |
| EA | $?$ | $?$ |

6 a. Define the terms: Level surface, Bench mark, Reduced level, Back sight.
(04 Marks)
b. Explain temporary adjustments of dumpy level.
(06 Marks)
c. The following staff readings were obseryed sticcessively with a level the instrument having been moved after third, sixth and eighth readings : $2.228,1.606,0.988,2.090,2.864,1.262$, $0.602,1.982,1.044,2.684$ meters. Calculate the RL's of all points if the RL of first reading was taken with a staff held on a BM of RL $432,384 \mathrm{~m}$. Adopt rise and fall method. ( 10 Marks)

7 a. Explain the characteristics of contour.
(06 Marks)
b. With neat sketch, explain (i) Profile levelling
(ii) Reciprocal levelling
(iii) Block levelling.
c. The following observations were taken in reciprocal levelling:

| Inst. at | Staff reading at |  |
| :---: | :---: | :---: |
|  | A | B |
| A | 1.625 | 2.545 |
| B | 0.725 | 1.405 |

Determine the KL of B , if that of A is 100.800 m . Also calculate the angular error in collimation if the distance between $A$ and $B$ is 1000 m .
(08 Marks)

8 a. Discuss the advantages and disadvantages of plane tabling.
(06 Marks)
b. Explain three point problem, by Bessel's graphical method.
(08 Marks)
c. Explain methods of orientation by plane table.


# Third Semester B.E. Degree Examination, June/July 2018 Fluid Mechanics 

Time: 3 hrs.
Max. Marks: 100
Note: Answer any FIVE full questions, seiecting at least TWO questions from each part.

## PART - A

1 a. Define the following. Also mention their units (i) Specific weight (iii) Specific volume (iv) Mass density.
(ii) Relative density
(08 Marks)
b. Two vertical parallel plates distance ' $t$ ' apart are partially submerged in liquid of specific weight ' $w$ ' and surface tension $\sigma$. Show that capillary rise is given by: $h=\frac{2 \sigma \operatorname{Cos} \theta}{t w}$ where $\theta$ is angle made by surface tension force with vertical.
(04 Marks)
c. An oil of viscosity 5 poise is used for lubrication between a shaft ad sleeves. The diameter of shaft is 0.5 m and it rotates at 200 rpm . Calculate the power lost in oil for a sleeve length of 100 mm the thickness of oil film is 1 mm .
(08 Marks)
2 a. Differentiate between: (i) Absolute pressure and gauge pressure (ii) Simple manometer and differential monometer and (iii) Piezometer and pressure gauges.
(06 Marks)
b. What is U-tube differential manometer? Obtain an expression for difference of pressure between two pipes at different levels.
(06 Marks)
c. A piezometer tube is fitted to a tank containing water at a point 500 mm above the bottom of tank as shown in Fig Q2(c). The liquid in manometer is carbon disulphide having a specific gravity 1.9. Find the height of free water surface above the bottom of tank if piezometer reading is 400 mm . find also pressure intensity at bottom of tank.
(08 Marks)


Fig Q2(c)
3 a. What is total pressure and centre of pressure? Explain.
(04 Marks)
b. Derive an expression for force excreted on submerged inclined plane surface by static liquid and locate the position of centre of pressure.
(06 Marks)
c. An inclined rectangular gate of width 5 m and depth 1.5 m is installed to control the discharge of water as show in Fig Q3(c). The end A is hinged. Determine the force normal to the gate applied at B to open it.


Fig Q3(c)
1 of 2
(10 Marks)

4 a. Differentiate between :
(i) Streamline and Streak line
(ii) Stream function velocity potential function
(iii) Uniform and Non Uniform flow
(iy) Rotational and Irrotational flow.
(08 Marks)
b. What are equipotential line and line of constant stream function? Show that they are orthogonal to each other?
(04 Marks)
c. The stream function in a two dimensional flow field is $\Psi=6 x-4 y+7 x y$. Verify whether the flow is irrotational. Determine the direction of stream line at point $(1,-1)$. Determine also expression for velocity potential.
(08 Marks)

## PART - B

5 a. Write Euler's equation of motion along a streamline and integrate it to obtain Bernoulli's equation. State also assumption made.
(10 Marks)
b. At a point the pipe line where the diameter is 20 cm , the velocity of water is $4 \mathrm{~m} / \mathrm{s}$ and pressure is $343 \mathrm{kN} / \mathrm{m}^{2}$. At a point 15 m downstream the diameter reduces to 10 cm . Calculate the pressure at this point if pipe is (i) horizontal (ii) vertical with flow downwards (iii) vertical with flow upwards.
(10 Marks)

6 a. Define Hydraulic Gradient Line and Total Energy Line. Explain with sketch.
(04 Marks)
b. Derive an expression for pressure rise due to sudden closure of valve when the pipe is elastic.
(08 Marks)
c. Two tanks are connected with help of two pipes in series. The lengths of pipes are 1000 m and 800 m where as the diameters are 400 mm and 200 mm respectively. The coefficient of friction for both the pipes is 0.008 . The difference of water level in two tanks is 15 m . Find the rate of flow of water through pipes, considering all losses.
(08 Marks)

7 a. Write Short notes on:
(i) Staff gauge
(ii) Weight gauge
(iii) Float gauge
(iv) Hook gauge
(08 Marks)
b. Explain the method of measurement of velocity by current meter.
(04 Marks)
c. A pitot tube records reading of 7.85 KPa as the stagnation pressure, when it is held at centre of pipe of 250 mm diameter conveying water. The static pressure pipe is 40 mm of mercury (Vacuum). Calculate the discharge in pipe assuming the mean velocity of flow is 0.8 times the velocity at centre. Take $\mathrm{C}_{\mathrm{v}}=0.98$.
(08 Marks)

8 a. Write a note on cippolletti weir.
(04 Marks)
b. Derive an expression for discharge through a venturimeter.
(08 Marks)
c. A venturimeter is installed in a pipeline 30 cm in diameter. The throat pipe diameter ratio is $\frac{1}{3}$. Water flows through instaltation. The pressure in the pipeline is $137.7 \mathrm{kN} / \mathrm{m}^{2}$ and vacuum in the throat is 37.5 cm of mercury. If $4 \%$ of differential head is lost between the gauges, find the flow in the pipe line.
(08 Marks)

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MATDIP301

Third Semester B.E. Degree Examination, June/July 2018
Advanced Mathematics - I
Time: 3 hrs .
Max. Marks: 100

## Note: Answer any FIVE full questions.

1 a. Find modulus and amplitude of : $z=\frac{(1+i)^{2}}{1-i}$.
(06 Marks)
b. Prove that :
$(1+\cos \theta+i \sin \theta)^{n}+(1+\cos \theta-i \sin \theta)^{n}=2^{n H} \cos ^{n} \frac{\theta}{2} \cos \frac{n \theta}{2}$
(07 Marks)
c. If $x=\cos \theta+i \sin \theta$ and $y=\cos \phi+i \sin \phi$, then prove that $\frac{x-y}{x+y}=i \tan \left(\frac{\theta-\phi}{2}\right)$.

2 a. Find the $n^{\text {th }}$ derivative of $y=e^{a x} \cos (b x+c)$
(06 Marks)
b. If $y=e^{m \sin ^{-1}} x$ then prove that $\left(1-x^{2}\right) y_{n+2}-(2 n+1) x y_{n+1}-\left(n^{2}+m^{2}\right) y_{n}=0$.
(07 Marks)
c. Expand $\log (1+\sin \mathrm{x})$ in powers of x , by using Maclaurin's theorem.
(07 Marks)

3 a. If $z=e^{a x+b y} f(a x-b y)$, then show that $b \frac{\partial z}{\partial x}+a \frac{\partial z}{\partial y}=2 a b z$.
(06 Marks)
b. If $u=\tan ^{-1}\left(\frac{x^{3}+y^{3}}{x-y}\right)$ then prove that $x \frac{\partial u}{\partial x}+y \frac{\partial u}{\partial y}=\sin 2 u$.
(07 Marks)
c. If $u=\tan ^{-1} x+\tan ^{-1} y$ and $v=\frac{x+y}{1-x y}$ find $\frac{\partial(u, v)}{\partial(x, y)}$
(07 Marks)

4 a. With usual notation, prove that $\tan \phi=\mathrm{r} \frac{\mathrm{d} \theta}{\mathrm{dr}}$.
(06 Marks)
b. Find the angle between the curves $r=a(1-\cos \theta)$ and $r=2 a \cos \theta$.
(07 Marks)
c. Find the pedal equation of the curver $=a(1+\cos \theta)$.
(07 Marks)

5 a. Obtain the reduction formula for $\int \sin ^{n} \mathrm{xdx}$, where n is a positive integer. (06 Marks)
b. Evaluate $\int_{0}^{1} \frac{x^{9}}{\sqrt{1-x^{2}}} \mathrm{dx}$.
(07 Marks)
c. Evaluate $\int_{0}^{\log 2} \int_{0}^{x+y} \int_{0}^{x+y+z} e^{x+y d y d x .}$
(07 Marks)

6 a. Prove that $\sqrt{\frac{1}{2}}=\sqrt{\pi}$.
b. Show that $\int_{0}^{\pi / 2} \sqrt{\sin \theta} \times \int_{0}^{\pi / 2} \frac{1}{\sqrt{\sin \theta}} d \theta=\pi$.
(06 Marks)
c. Evaluate $\int_{0}^{\infty} \frac{\mathrm{dx}}{1+\mathrm{x}^{4}}$ in terms of Beta functions.
(07 Marks)
(07 Marks)

7 a. Solve $\frac{d y}{d x}=\sin (x+y)$.
(06 Marks)
b. Solve $x d y-y d x=\sqrt{x^{2}+y^{2}} d x$.
c. Solve $\left(x^{2}-4 x y-2 y^{2}\right) d x+\left(y^{2}-4 x y-2 x^{2}\right) d y=0$.
$8 \quad$ Solve $\frac{d^{3} y}{d x^{3}}-6 \frac{d^{2} y}{d x^{2}}+11 \frac{d y}{d x}-6 y=0$.
b. Solve $\frac{d^{2} y}{d x^{2}}-4 \frac{d y}{d x}+4 y=e^{2 x}+\cos 2 x$.
(06 Marks)
(07 Marks)
c. Solve $\frac{d^{2} y}{d x^{2}}-2 \frac{d y}{d x}+2 y=e^{x} \cos x$.

